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Since a tangent from a point to a parabola makes the same angle with the line joining the point to the focus as the remaining tangent from the point does with the axis, we have $\angle TPA=\angle QPS=\angle T'PA'$. Hence $\angle TPT'=\angle APA'$.

II. **Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.**

Let $y^2=4a(x+a)$.....(1),

$y^2\cos^2\theta+2y\sin\theta(x\cos\theta+2b)=4bx\cos\theta+4b^2-x^2\sin^2\theta$.....(2),

be the confocal parabolae with axes inclined at an angle θ.

Let the common tangent be $y=mx+c$.....(3).

If (3) is tangent to (1), $c=a(m^2+1)/m$.

If (3) is tangent to (2), $c=b(m^2+1)/(m\cos\theta+\sin\theta)$.

$$\therefore\ m=\frac{a\cos\theta}{b-a\cos\theta},\quad c=\frac{a^2+b^2-2ab\cos\theta}{\sin\theta(b-a\cos\theta)}.$$

$$\therefore\ (3)\text{ becomes } y=\frac{ax\sin\theta}{b-a\cos\theta}+\frac{a^2+b^2-2ab\cos\theta}{\sin\theta(b-a\cos\theta)}\text{.....(4)}.$$

Let $(h,\ k)$ be the point on (4).

$\therefore\ y=m_1x+a(m_1^2+1)/m_1$ is tangent to (1),

$y=m_2x+b(m_2^2+1)/(m_2\cos\theta+\sin\theta)$ is tangent to (2).

But $k=m_1h+a(m_1^2+1)/m_1$.....(5),

$k=m_2h+b(m_2^2+1)/(m_2\cos\theta+\sin\theta)$.....(6),

$$k=\frac{ah\sin\theta}{b-a\cos\theta}+\frac{a^2+b^2-2ab\cos\theta}{\sin\theta(b-a\cos\theta)}\text{.....(7)}.$$

$$\text{From (5), } m_1^2-\frac{km_1}{a+h}+\frac{a}{a+h}=0\text{.....(8)}.$$

$$\text{From (6), } m_2^2+\frac{(h\sin\theta-k\cos\theta)m_2}{h\cos\theta+b}+\frac{b-k\sin\theta}{h\cos\theta+b}=0\text{.....(9)}.$$

$\therefore\ m_1$ and m_2 each has two values.

From (4), $m_1=m_2=a\sin\theta/(b-a\cos\theta)$. The other values of m_1 and m_2 are respectively,

$$\frac{b-a\cos\theta}{(a+h)\sin\theta},\quad \frac{(b-k\sin\theta)\,(b-a\cos\theta)}{a\sin\theta(h\cos\theta+b)}=\frac{b\cos\theta-h\sin^2\theta-a}{\sin\theta(h\cos\theta+b)}.$$

$$\therefore\ \frac{m_1-m_2}{1+m_1m_2}=\frac{(a^2+b^2-2ab\cos\theta+2ah\sin^2\theta+h^2\sin^2\theta)\sin\theta}{(a^2+b^2-2ab\cos\theta+2ah\sin^2\theta+h^2\sin^2\theta)\cos\theta}=\tan\theta.$$

209. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

Find by a geometrical method the maximum value of $\sin\theta\cos\theta\cos2\theta$.

Solution by G. W. DRAKE, Fayetteville, Ark.

In the circle, radius 1 and center O, let $\angle AOB=\theta$, and $\angle AOC=2\theta$. From C drop a perpendicular to AO or AO produced, meeting AO in E. Then $CE=\sin2\theta$, and $OE=\cos2\theta$. $\sin\theta\cos\theta\cos2\theta=\frac{1}{2}\sin2\theta\cos2\theta=$the area of triangle OEC. But triangle OEC is a maximum when $OE=CE$. Hence $\sin\theta\cos\theta\cos2\theta$ is a maximum when $\sin2\theta=\cos2\theta$, *i. e.* when $2\theta=90-2\theta$, or when $\theta=22\frac{1}{2}°$.

Also solved by L. E. Newcomb, Los Gatos, Cal.; G. B. M. Zerr, A. M., Ph. D., Parsons, West Va. [Dr. Zerr gives the more general result $\theta=\frac{1}{8}\pi(4m+1)$.]

210. Proposed by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.

Let ADC be a triangle with angle $C=120°$, and let the interior bisector of angle C meet AD in B. Prove that $2.CB$ is the harmonic mean between CA and CD.

Solution by M. E. GRABER, A. B., Instructor in Mathematics and Physics, Heidelberg University, Tiffin, O., and G. W. DRAKE, Fayetteville, Ark.

On AC produced through C, take a distance $CK=CD$, and join K and D. Since triangle ACB is similar to triangle AKD, $\therefore$ $BC{:}DK=CA{:}KA$, hence $2.BC=\dfrac{2.CA.DK}{KA}=\dfrac{2.CA.DK}{CA+KC}$. But because $CD=CK$, and $\angle KCD=60°$, $\therefore \angle CKD=\angle KDC=60°$, and triangle CKD is equilateral. $\therefore$ $2.BC=\dfrac{2CA.CD}{CA+CD}$. Hence $2.BC$ is the harmonic mean between CA and CD by definition.

Also solved by R. A. Wells, Bellevue College, Bellevue, Nebr.; G. W. Greenwood, B. A. (Oxon), Professor of Mathematics and Astronomy, McKendree College, Lebanon, Ill.; L. E. Newcomb, Los Gatos, Cal.; G. B. M. Zerr, A. M., Ph. D., Parsons, W. Va.; J. Scheffer, Kee Mar College, Hagerstown, Md.; E. L. Sherwood, Shady Side Academy, Pittsburg, Pa.

211. Proposed by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.

Prove the validity of the following construction of an inscribed regular pentagon and regular decagon: Draw any two perpendicular radii of the given circle with center C. Call E the end of one radius CE and M the middle point of the perpendicular radius CM. Take the point R on CM produced through C such that $RCM=EM$. Then $RC=$side of inscribed regular decagon, $RE=$side of inscribed regular pentagon.

Solution by G. W. DRAKE, Fayetteville, Ark., and R. A. WELLS, Bellevue College, Bellevue, Neb.

Join E and M, also E and R. Let $r=CE$.

(1). $ME^2=\frac{1}{4}r^2+r^2=5r^2/4$. $ME=\frac{1}{2}r\sqrt{5}$.

$\therefore RC=RM-CM=ME-CM=\frac{1}{2}r\sqrt{5}-\frac{1}{2}r=\frac{1}{2}r(\sqrt{5}-1)=$a side of a regular decagon inscribed in a circle whose radius is r.

(2). $RE^2=RC^2+CE^2=[\frac{1}{2}r(\sqrt{5}-1)]^2+r^2=\frac{1}{4}r^2(6-2\sqrt{5})+r^2=\frac{1}{4}r(10-2\sqrt{5})$. $\therefore RE=\frac{1}{2}r\sqrt{(10-2\sqrt{5})}=$a side of a regular pentagon inscribed in a circle whose radius is r.

Also solved by G. W. Greenwood, A.B. (Oxon), Professor of Mathematics and Astronomy, McKendree College, Lebanon, Ill.; L. E. Newcomb, Los Gatos, Cal.; G. B. M. Zerr, A. M., Ph. D., Parsons, W. Va.; J. Scheffer, Kee Mar College, Hagerstown, Md., G. I. Hopkins, Manchester, N. H.